Charge dependence and electric quadrupole effects on single-nucleon removal in relativistic and intermediate energy nuclear collisions

John W. Norbury
Department of Physics, Rider College, Lawrenceville, New Jersey 08648
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Single-nucleon removal in relativistic and intermediate energy nucleus-nucleus collisions is studied using a generalization of Weizsäcker-Williams theory that treats each electromagnetic multipole separately. Calculations are presented for electric dipole and quadrupole excitations and incorporate a realistic minimum impact parameter, Coulomb recoil corrections, and the uncertainties in the input photonuclear data. Discrepancies are discussed. The maximum quadrupole effect to be observed in future experiments is estimated and also an analysis of the charge dependence of the electromagnetic cross sections down to energies as low as 100 MeV/nucleon is made.

There has recently been considerable interest in single-nucleon removal in nucleus-nucleus reactions.[1–14] A large part of the cross section is due to electromagnetic (em) excitations which should be easily calculable by the Weizsäcker-Williams (WW) method[9] or by a simple generalization which distinguishes between electric multipoles.[4,8–11] Unfortunately, recent theoretical comparisons[3–5] to Bevalac data[7] indicated several discrepancies. Benesh, Cook, and Vary[3] have suggested that these discrepancies could be due to difficulties in subtracting the nuclear component from the total measured cross section. These authors also addressed the problem of what value to use for the minimum impact parameter which has been independently verified[12] to within a few percent. Even though the cross-section calculations of Benesh, Cook, and Vary look very promising, problems remain with ^{59}Co and ^{197}Au at energies relevant to the European Center for Nuclear Research (CERN).

A new experimental technique which attempts to avoid the above problem has been developed,[1] and new data are now available for nucleon removal from ^{28}Si at Alternating Gradient Synchrotron (AGS) energies. Other interesting work[2] has also been done on the charge dependence of the various processes in nucleus-nucleus reactions.

It is very important to fully understand em processes in nuclear collisions for all energies and all nuclei. The WW method has proven to be a useful tool in this context, but a more accurate theoretical analysis (herein referred to as "multipole theory") was developed by Bertulani and Baur,[9] Fleischhauer and Scheid,[10] and Goldberg[11] which treats each electric multipole separately.

The present paper is a continuation of previous work which used this more accurate analysis in understanding recent data.[4] The new items to be studied herein are as follows:

(1) inclusion of Coulomb recoil,[8] which Aleixo and Bertulani have shown enables the multipole theory of the virtual photon spectra[9] to be used with confidence for energies as low as 100 MeV/nucleon where the WW method breaks down;

(2) comparison of the multipole theory to new data at low energy[13] (150 MeV/nucleon), ^{28}Si data[1] at AGS energies (14.6 GeV/nucleon), and ^{32}S data[14] at CERN energy (200 GeV/nucleon);

(3) inclusion of experimental uncertainties in the photonuclear data, which is used as input into the multipole theory to arrive at a theoretical error giving better guidance in comparison to data;

(4) estimates of the maximum effect of electric quadrupole ($E2$) components in future experiments; and

(5) extension of recent WW studies of charge dependences[2] to much lower energies using multipole theory.

As pointed out in Refs. 4 and 9, the *isoscalar* component of the giant quadrupole resonance (GQR) and the isovector giant dipole resonance are expected to dominate single-nucleon removal cross sections. The isovector GQR lies at higher energy, where the virtual photon spectrum is much smaller, and decays mainly by two-nucleon emission. Note further that $E2$ transitions do *not* have the isospin selection rules found for $E1$ transitions.

The dipole and quadrupole cross sections discussed are calculated according to the method of Ref. 4 using the minimum impact parameter of Ref. 3 [which is expected to be more accurate[3,12] than the $1.2(A_P^{1/3}+A_T^{1/3})$ parametrization] and with the addition of the intermediate energy Coulomb recoil correction $\pi a_0/2\gamma$ of Ref. 8. [Note that there is a typing error in the first paper of Ref. 4. Equation (4) in that reference should have E_{GQR}^2 in the numerator and not E^2. Also, in Table I of Ref. 5, the last entry in the fifth column should read 335±49 and not 73±13.]

Quadrupole parameters are listed in Ref. 4 except those for ^{238}U and ^{28}Si for which the energy (MeV), width (MeV), and fractional exhaustion, respectively, are 10.2, 2.5, and 0.85 for ^{238}U and 19.7, 5.1, and 0.2 for ^{28}Si. The theoretical uncertainties based upon the uncertainties of the experimental photonuclear cross sections used as input are estimated to be 10% for ^{28}Si and ^{12}C and 5% for the heavier nuclei. The input photonuclear data are discussed in Ref. 5, and in Ref. 15 for ^{238}U and ^{28}Si. In addi-

TABLE I. Calculated cross sections $\sigma_{E1}+\sigma_{E2}$, which include the intermediate energy recoil correction of Ref. 8 and the impact parameter of Ref. 3, are added to $\sigma_{\rm nuclear}$ (Ref. 3) and compared to the total experimental cross sections of Ref. 7. The 150-MeV/nucleon data are from Ref. 13. All results refer to single-neutron removal from the target. See Ref. 16 for an important note.

Projectile	Target	$T_{\rm lab}$ (GeV/nucleon)	σ_{E1} (mb)	σ_{E2} (mb)	$\sigma_{\rm nuclear}$ (mb)	$\sigma_{E1}+\sigma_{E2}+\sigma_{\rm nuclear}$ (mb)	$\sigma_{\rm total}^{\rm expt}$ (mb)
^{12}C	^{238}U	2.1	29	9	132	170±8	173±22
^{20}Ne	^{238}U	2.1	78	22	140	240±12	192±16
^{12}C	^{197}Au	2.1	38	7	124	169±8	178±7
^{20}Ne	^{197}Au	2.1	102	17	131	250±14	268±11
^{40}Ar	^{197}Au	1.8	286	48	142	476±34	463±30
^{56}Fe	^{197}Au	1.7	558	93	149	800±66	707±52
^{139}La	^{197}Au	1.26	2008	357	169	2534±237	2130±120
^{139}La	^{197}Au	0.15	574	566	177	1317±114	765±48
^{16}O	^{197}Au	60	211	15	128	354±24	400±20
^{16}O	^{197}Au	200	273	17	128	418±30	560±30
^{12}C	^{89}Y	2.1	13	1	98	112±5	115±6
^{20}Ne	^{89}Y	2.1	34	3	105	142±6	160±7
^{40}Ar	^{89}Y	1.8	94	9	115	218±12	283±11
^{56}Fe	^{89}Y	1.7	181	17	121	319±21	353±14
^{12}C	^{59}Co	2.1	8	1	87	96±5	89±5
^{20}Ne	^{59}Co	2.1	20	1	94	115±5	132±7
^{56}Fe	^{59}Co	1.7	103	7	110	220±13	194±9
^{139}La	^{59}Co	1.26	351	26	129	506±38	450±30
^{12}C	^{12}C	2.1	0.5	0	59	60±3	60.7±0.6
^{20}Ne	^{12}C	1.05	1	0	66	67±4	78±2
^{56}Fe	^{12}C	1.7	6	0	83	89±5	94±2
^{139}La	^{12}C	1.26	20	1	102	123±7	148±2

tion, a 5% error is included for possible uncertainties occurring in the $E2$ parameters.[4] Unlike previous work, the present electromagnetic multipole cross sections are added to the nuclear cross sections of Benesh, Cook, and Vary[3] (see Ref. 16 for an important note) and compared to the originally measured total cross section. The final theoretical uncertainty in $\sigma_{E1}+\sigma_{E2}+\sigma_{\rm nuclear}$ incorporates the errors discussed above together with the theoretical uncertainties of Ref. 3. In addition, the em calculations are compared to newly published em data[1,13,14] for energies ranging from 150 MeV/nucleon to 14.6 GeV/nucleon to 200 GeV/nucleon (see Table I).

Taken together with previous comparisons,[1,3–5,7] the results shown in Tables I and II show substantial improvement in understanding the data. The apparent disagreements between theory and experiment for ^{28}Si+^{27}Al are discussed in Ref. 1 as likely due to a remaining hadronic component. Excellent agreement between theory and experiment is obtained for ^{32}S+^{197}Au at 200 GeV/nucleon even though poor agreement is obtained for the ^{16}O projectile at the same energy. An additional measurement seems in order here. Some of the other disagreements such as ^{40}Ar+^{89}Y and ^{139}La+^{12}C may be due to uncertainties in the nuclear part of the cross section.[12] The

TABLE II. Calculated cross sections, as in Table I, are compared to experimental em cross sections of Ref. 13 (^{139}La) and Ref. 14 (^{32}S). In the case of ^{28}Si (Ref. 1) the protons or neutrons are emitted from the projectile and only the experimental semi-inclusive cross sections are listed. All energies represent total energy E, except for 0.15 GeV/nucleon (first row), which represents kinetic energy T. The experimental numbers for ^{28}Si were obtained by adding up the exclusive cross sections listed in Ref. 6.

Projectile	Target	Lab energy (GeV/nucleon)	Final state	$\sigma_{\rm em}^{\rm expt}$ (mb)	$\sigma_{\rm WW}$ (mb)	σ_{E1} (mb)	σ_{E2} (mb)	$\sigma_{E1}+\sigma_{E2}$ (mb)
^{139}La	^{197}Au	0.15	^{196}Au	447	603±30	574	566	1140±114
^{32}S	^{197}Au	200	^{196}Au	1120±160	1104±55	1073	60	1133±113
^{28}Si	^{27}Al	14.6	$1p$	37±5	24±2	24	0	24±4
^{28}Si	^{27}Al	14.6	$1n$	15±4	9±1	9	0	9±1
^{28}Si	^{120}Sn	14.6	$1p$	313±4	317±32	315	3	318±48
^{28}Si	^{120}Sn	14.6	$1n$	136±6	118±12	118	1	119±18
^{28}Si	^{208}Pb	14.6	$1p$	743±27	806±81	802	8	810±122
^{28}Si	^{208}Pb	14.6	$1n$	347±18	301±30	299	2	301±45

TABLE III. Calculated cross sections, as in Tables I and II, for single-neutron emission from ^{197}Au targets at a variety of laboratory and center-of-mass (c.m.) energies. The relevant accelerators are listed in parentheses. Even though ^{197}Au projectiles will not be available at all the energies below, the same ^{197}Au projectile was used simply for the sake of comparison to provide an upper limit for the importance of $E2$ effects. (AGS: alternating gradient synchrotron; RHIC: relativistic heavy ion collider.) The last column represents $(\sigma_{E1}+\sigma_{E2}-\sigma_{WW})/(\sigma_{E1}+\sigma_{E2})$ as a (rounded-off) percentage. All cross sections are in units of barn.

Energy	σ_{WW}	σ_{E1}	σ_{E2}	$\sigma_{E1}+\sigma_{E2}$	Percentage difference
$T_{lab}=100$ MeV/nucleon	0.56	0.53	0.83	1.36	60%
$T_{lab}=300$ MeV/nucleon	1.7	1.6	0.8	2.4	30%
$T_{lab}=500$ MeV/nucleon	2.3	2.2	0.7	2.9	20%
$T_{lab}=2.1$ GeV/nucleon (Bevalac)	4.9	4.7	0.6	5.3	8%
$E_{lab}=12$ GeV/nucleon (AGS)	11.1	10.8	0.7	11.5	3%
$T_{lab}=60$ GeV/nucleon (CERN)	19.3	18.7	1.0	19.7	2%
$T_{lab}=200$ GeV/nucleon (CERN)	25.5	24.7	1.1	25.8	1%
$T_{lab}=200$ GeV/nucleon (RHIC)	49.5	48.1	1.4	49.5	0%

disagreements at 150 MeV/nucleon for ^{139}La+^{197}Au are discouraging since the new theoretical additions in the present work should be more significant at lower energies (Loveland *et al.*[13] have also recognized this discrepancy). Since there is only one data point at lower energy, further experiments between 100 MeV/nucleon and 1 GeV/nucleon are particularly welcome. Finally, some light may be shed on the ^{139}La+^{197}Au disagreement by the study of ^{197}Au+^{197}Au at AGS energies.

To serve as a guide for the relative importance of $E2$ effects, the percentage differences between WW and multipole theory cross sections are shown in Table III. Calculations are presented for nucleon emission from ^{197}Au, which has one of the largest giant quadrupole resonances. Thus the cross sections listed represent the *maximum* $E2$ effect that one is ever likely to observe at the selected energies. A negligible percentage difference means that one would get just as good results using WW theory rather than multipole theory. As expected, $E2$ effects are not really relevant for energies above that of the AGS. *Note most importantly that this conclusion is only valid for single-nucleon emission.* Two-neutron removal may well be observed at high energy due to decay of the isovector giant quadrupole resonance.

Finally, the charge dependence of em processes in nucleus-nucleus collisions has been previously described by Hill *et al.*[7] and Lissauer and Takai,[2] who note that significant deviations from a simple Z^2 dependence can occur in the WW formalism. However, the WW method is limited to high energies and the advantage of the multipole theory incorporating recoil corrections is that the charge dependence studies can be taken to much lower energies. In Fig. 1 the cross section is plotted versus the charge of the incident nucleus. Note that the log plot

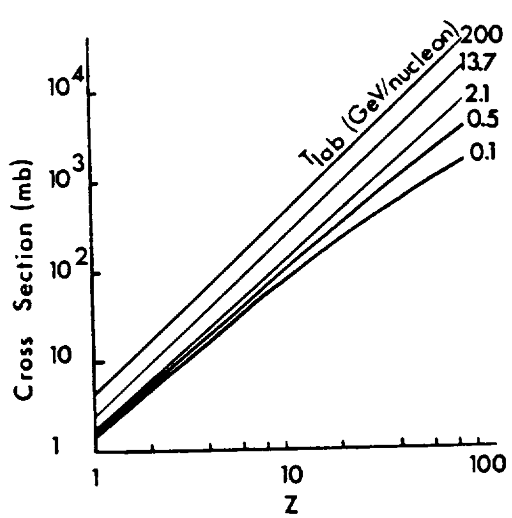

FIG. 1. ^{197}Au neutron removal cross section (mb) versus nuclear charge as a function of projectile energy.

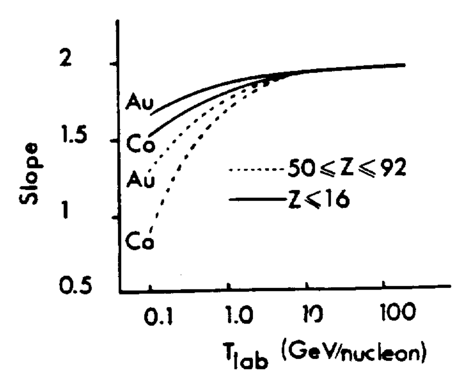

FIG. 2. Power of Z dependence (slope) versus projectile energy for neutron removal from ^{197}Au and ^{59}Co. The solid lines represent the low-Z region ($Z \le 16$) and the dashed lines represent high-Z region ($50 \le Z \le 92$). The dashed lines merge with the solid lines at about 10 GeV/nucleon.

are *curved* (particularly at the lower energies), indicating that there is no unique Z dependence. Nevertheless, a straight line can be fitted to the low-Z ($Z \leq 16$) region and a line of a different slope can be fitted to the high-Z ($50 \leq Z \leq 92$) region. At high energy these lines become indistinguishable from one another. To illustrate Z dependence, plots similar to Ref. 2 are shown in Fig. 2 for both high- and low-Z for single-neutron removal from ^{59}Co and ^{197}Au. Clearly, it is not possible to average out the curves into a single curve. Furthermore, one expects[2] that for processes corresponding to different photonuclear energies the corresponding plots would not overlap those of Fig. 2.

Even though one should not extend WW theory to lower energies, nevertheless if one does this, then the WW plots corresponding to Fig. 1 come out with *exactly the same shapes*, although the cross sections are all smaller. *Thus Fig. 2 is identical for both WW and multipole theory.*

In summary, the electromagnetic multipole theory[9] for nucleon emission from nucleus-nucleus reactions incorporating realistic minimum impact parameter,[3,12] Coulomb recoil correction,[8] and photonuclear data and quadrupole parameter uncertainties has been added to nuclear interaction cross sections[3] and compared to previous[7] and new data.[1,13,14] The maximum amount of $E2$ contribution has been noted and experimental discrepancies pinpointed. An analysis of charge dependence of the cross section down to energies as low as 100 MeV/nucleon has also been made.

Note added in proof. It should be noted that the $E2$ sum rule (which is strictly only valid for spin-zero nuclei) used in Refs. 4 and 9 and the present work has not been separated into its individual isoscalar and isovector parts. The reason for this is discussed by E. C. Halbert, J. B. McGrory, G. R. Satchler, and J. Speth in Nucl. Phys. **A245**, 189 (1975), where they show that the usual method of multiplying the sum rule by Z/A (to obtain the isoscalar component) can lead to an overexhaustion of the sum rule for non-self-conjugate nuclei. Given this, and the fact that most of the nuclei in Refs. 4 and 9 and the present work are not spin zero, the magnitude of the calculated electric quadrupole effects should be considered as an *approximate upper limit.*

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